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9 FINAL REPORT

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ABSTRACT

This final report briefly summarizes the development and application of approximation and bounding techniques to nonlinear diffusion equations modelling phenomena in lubrication theory, combustion theory, heat flow, etc. The development of mathematical techniques has been guided by the need to meet physical problems. The mathematical techniques have been used to illuminate the behavior of models of various nonlinear diffusion phenomena. Techniques include monotone approximation schemes and nonlinear comparison theorems. These permit, for example, deriving bounds on solutions of nonlinear diffusion equations by finding functions which satisfy appropriate sets of differential inequalities. A typical application of these techniques has been the exploration of the relation between the stationary approximation of combustion theory and the full, time-dependent model.

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The research supported by the grant for which this is the final report has dealt with a number of topics relating to the construction and application of a priori bounds for nonlinear diffusion problems. While some of the approximation techniques developed suggest computation schemes that could be implemented numerically, the emphasis of this study has been on the use of approximations and bounds to obtain qualitative information which illuminates the behavior of mathematical models of nonlinear diffusion phenomena.

An important tool in these studies has been monotone approximation methods, which construct maximal and minimal solutions of nonlinear diffusion problems as limits of monotone sequences beginning from upper and lower solutions. In essence, this procedure permits one to construct bounds on relatively complicated nonlinear equations by finding functions which satisfy the equations merely in the sense of inequality. (The direction of the inequality determines whether the approximate solution is an upper bound or lower bound.)

A manuscript [1] prepared with the support of this grant shows that a monotone approximation scheme for steady-state problems corresponds to the temporal evolution of that steady-state from an appropriate initial condition. Establishing the connection between the mathematical construction and the physical evolution of the steady-state helps to explain the success of the monotone approximation methods and the origin of the bounds which they construct.

Recent research, as yet incomplete, has shown that these monotone constructions and the resulting bounds can be extended to a very wide class of nonlinear problems. The essential ingredients appear to be a nonlinear comparison theorem and sufficient smoothness to force convergence of the monotone sequences. While the monotone schemes are not truly constructive, as are those mentioned above, they do promise to extend the idea of generating bounds by solving problems involving inequalities to a much wider class of problems.

An important mathematical tool in this approach is a comparison theorem or maximum principle which permits one to compare the unknown solution of the given problem with known functions that have been suggested as bounds by, say, physical arguments. A comparison theorem has been developed for an important class of operators which arise, for example, in hydrodynamic lubrication theory and heat flow studies in which the dependence of the thermal diffusion coefficient on temperature and/or its gradient must be included [2]. This result shows that the comparison theorem arises in a reasonably natural way from the structure of the operator, which is dictated by the physical diffusion phenomena, with a minimum of artificial mathematical restrictions.

The development and application of these ideas to scalar nonlinear diffusion equations is reviewed and summarized in a manuscript now in preparation [3].

A second partially completed manuscript [4] describes in detail the qualitative information one may extract with these techniques from the Reynolds equation of gas-lubrication theory. Some of the behavior described there casts doubt on certain aspects of the fundamental model of lubrication theory.

The problem of criticality in combustion theory has led to the development of comparison theorems for systems of nonlinear diffusion equations [5]. This paper points out that comparison theorems for systems of equations ultimately rely on an appropriate theorem for scalar equations. We endeavor to provide a "recipe" which would enable workers in other fields to construct system comparison results suitable for the problems they encounter from known scalar comparison results.

The comparison theorems developed there have been applied to explore the phenomena of criticality in combustion theory [6], [7]. The first article develops spatially uniform bounds while the second describes bounds which depend on both space and time. The process of constructing these bounds has a very simple physical interpretation which itself points out how these bounds can be improved. In essence, the bounding process consists of running an imaginary reaction at extremes of temperature and reactant concentration. The resulting reactant and temperature versus time curves can be shown to be rigorous bounds on the reactant and temperature histories of the actual model reaction.

Specifically, we have been able to demonstrate that the so-called Frank-Kamenetskii approximation produces a temperature field which dominates the true temperature point by point in the reaction vessel at each instant of time. Hence, criticality parameters predicted through the Frank-Kamenetskii approximation are lower bounds on reasonably defined criticality parameters for the actual model reaction. That is, the actual "cooler" model reaction cannot reach criticality before the "hotter" approximate reaction.

We believe that additional study of this model using comparison techniques will further elucidate underlying mechanisms and relations with approximations widely used in the combustion literature.

During the course of this research, it has also become apparent that many of the comparison theorems in the mathematical literature, which appear to have developed in a rather haphazard manner, could perhaps be derived by a common method. When these ideas have been developed more fully and prepared for publication, appropriate acknowledgment for ARO support will be given.

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